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On Regular and Intra-Regular Γ-Semihyperrings

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Abstract

The Γ -semihyperring is a generalization of the concepts of a semiring, a semihyperring and a Γ -semiring. In this paper we studied regular and intra-regular Γ -semihyperring and we have made the characterization of regular and intra-regular Γ -semihyperring with the help of different types of ideals in the Γ -Semihyperring.

Keywords: Quasi-ideal, Bi-ideal, Regular Γ-Semihyperring.

1. Introduction

In mathematics theory of classical algebraic structure and theory of hyperstructure these are two approaches to study the various concepts. In theory of classical algebraic structure the composition of two elements is an element and in theory of hyperstructure the composition of two element becomes a set. The mathematicians was well known to the concept of classical algebraic structure and various concepts was being studied in this respect. In 1934, the concept of hyperstructure theory comes before the word very first when French Mathematician presented paper in conference. When it is found that the theory of hyperstructure has vast application in various branches of science and then theory of hyperstructure becomes popular and being studied by mathematicians across the word. In 2003, Corsini and Leoreanu [1] have given application of theory of hyperstructures in various subjects like: geometry, cryptography, artificial intelligence, relation algebras, automata, median algebras, relation algebras, fuzzy sets and codes. If we let H be a non-empty set. Then, the map $o : H \times H \longrightarrow P^*(H)$ is called a hyperopertion, where $P^*(H)$ is the family of all non-empty subsets of H and the couple (H, o) is called a hypergroupoid. Moreover, the couple (H, o) is called a semihypergroup if for every $a, b, c \in H$ we have, $(aob)o\ c = ao(boc)$.

As theory of hyperstructure has vast application in various fields of sciences so it is essential to study the concepts of classical algebraic structure in hyperstructure theory. The main aim of this paper is to study the concepts of classical algebraic structure to a hyperstructure theory. Jagatap and Pawar [2] introduced intra-regular Γ-Semiring and made its characterizations with the help of ideals Γ-Semirings. Pawar et al. [3] introduced the notion of regular Γ-semihyperring. Patil and Pawar [4, 5] introduced and studied quasi-ideals and bi-ideals of a Γ-semihyperring. Here the concept of intra-regular Γ-Semihyperring is introduced and made its characterizations with the help of ideals, bi-ideals and quasi-ideals of Γ-Semihyperrings analogues to Jagatap and Pawar [2]. Ostadhadi-Dehkordi and Davvaz [6] studied ideal theory in Γ-Semihyperrings.

In this paper regular and intra-regular Γ-Semihyperring has been studied on the line of Jagatap and Pawar [2]. In section 2, Preliminaries are given required to understand the paper. In section 3, we have characterized regular and intra-regular Γ-Semihyperring analogues to [2]

2. Preliminaries

Here are some useful definitions and the readers are requested to refer [6-7].

Definition 2.1. [6] Let R be a commutative semihypergroup and Γ be a commutative group. Then, R is called a Γ-semihyperring if there is a map $R \times \Gamma \times R \longrightarrow P^*(H)$ (images to be denoted by $a\alpha b$, for all $a, b \in R$ and $\in \Gamma$) and $P^*(R)$ is the set of all non-empty subsets of R satisfying the following conditions:

(1) $a\alpha(b + c) = a\alpha b + a\alpha c$.

(2) $(a + b)\alpha c = a\alpha c + b\alpha c$.

(3) $a(\alpha + \beta)c = a\alpha c + a\beta c$.

(4) $a\alpha(b\beta c) = (a\alpha b)\beta c$, for all $a, b, c \in R$ and for all $\alpha, \beta \in \Gamma$.

Definition 2.2. [6] A Γ-semihyperring R is said to be commutative if $a\alpha b = b\alpha a$ for all $a, b \in R$ and $\alpha \in \Gamma$.

Definition 2.3. [6] A Γ-semihyperring R is said to be with zero, if there exists $0 \in R$ such that $a \in a + 0$ and $0 \in 0\alpha a$, $0 \in a\alpha 0$ for all $a \in R$ and $\alpha \in \Gamma$.

Let A and B be two non-empty subsets of a Γ-semihyperring R and $x \in R$, then

$$A + B = \{x | x \in a + b, a \in A, b \in B\}$$

$$A\Gamma B = \{x \,| x \in a\alpha b, a \in A, b \in B, \alpha \in \Gamma\}$$

Definition 2.4. [6] A non-empty subset $R_1$of Γ-semihyperring R is called a Γ -subsemihyperring if it is closed with respect to the multiplication and addition, that is, $R_1 + R_1 \subseteq R_1$ and $R_1\Gamma R_1 \subseteq R_1$.

Definition 2.5. [6] A right (left) ideal I of a Γ-semihyperring R is an additive sub semihypergroup of $(R, +)$ such that $I\Gamma R \subseteq I$ $(R\Gamma I \subseteq I)$. If I is both right and left ideal of R, then we say that I is a two sided ideal or simply an ideal of R.

Definition 2.6. [2] A non-empty set B of Γ-semihyperring R is a bi-ideal of R if B is a Γ-subsemihyperring of R and $B\Gamma R\Gamma B \subseteq B$.

Definition 2.7. [4] A subsemihypergroup Q of $(R, +)$ is said to be a quasi-ideal of Γ -semihyperring R if $(R\Gamma Q) \cap (Q\Gamma R) \subseteq Q$.

Definition 2.8. [2] An element e of Γ-semihyperring R is said to be a left (right) identity of R if $r \in e\alpha r$ $(r \in r\alpha e)$ for all, $r \in R$ and $\alpha \in \Gamma$.

An element e of Γ-semihyperring R is said to be a two sided identity or simply an identity if e is both left and right identity, that is $r \in e\alpha r \cap r\alpha e)$ for all $r \in R$ and $\alpha \in \Gamma$

Theorem 2.9. [4] Every quasi-ideal of a Γ-Semihyperring R is a bi-ideal of R.

Theorem 2.10. [5] Every one sided (two sided) ideal of a Γ-Semihyperring R is a bi-ideal of R.

Theorem 2.11. [4] Every one sided (two sided) ideal of a Γ-Semihyperring R is a quasi-ideal of R.

Definition 2.12. [8] A Γ-Semihyperring S is said to be an intra-regular Γ-Semihyperring if for any $x \in S$, $x \in S\Gamma x\Gamma x\Gamma S$.

Theorem 2.13. [8] Let S be a Γ-Semihyperring. Then S is intra-regular if and only if each right ideal R left ideal L of S satisfies $R \cap L \subseteq L\Gamma R$.

Definition 2.14. [2] A Γ-Semihyperring S is said to be a regular Γ-Semihyperring if for any $x \in S$, $x \in x\Gamma S\Gamma x$.

Theorem 2.15. [2] Let S be a Γ-Semihyperring with an identity. Then S is regular if and only if each right ideal R left ideal L of S satisfies $R \cap L = R\Gamma L$.

Theorem 2.16. [4] Intersection of left ideal and right ideal of a Γ-Semihyperring S is a quasi-ideal of S.

3. Regular and Intra-regular Γ-Semihyperring

In this section, we studied regular and intra-regular Γ-Semihyperring and made its characterization with the help of different ideals of Γ- Semihyperring on the line of Jagatap and Pawar [2]. Throughout this paper we consider that Γ-Semihyperring has identity element.

Theorem 3.1. Following statements are equivalent in Γ-Semihyperring S.

1. S is regular.
2. For any bi-ideal B of S, $B = B\Gamma S\Gamma B$.
3. For each quasi-ideal Q of S, $Q = Q\Gamma S\Gamma Q$

Proof. (1) ⇒ (2)

Suppose S is a regular Γ-Semihyperring and B be a bi-ideal of S. Then we have, $B\Gamma S\Gamma B \subseteq B$. For reverse inclusion let $b \in B$, then $b \in b\Gamma S\Gamma b \subseteq B\Gamma S\Gamma B$. Hence w get, $B = B\Gamma S\Gamma B$.

(2) ⇒ (3) As every quasi-ideal is bi-ideal by Theorem 2.9. Implication follows easily.

(3) ⇒ (1) Let R be a right-ideal and L be a left ideal of a Γ-Semihyperring S. By Theorem 2.16. $R \cap L$ Is a quasi-ideal of S. By (3), we have $R \cap L = (R \cap L)\Gamma S\Gamma(R \cap L) \subseteq R\Gamma S\Gamma L \subseteq R\Gamma L$. But $R\Gamma L \subseteq R \cap L$ always holds. Thus we get, $R \cap L = R\Gamma L$. Thus we get S is a regular Γ-Semihyperring by Theorem 2.15.

Theorem 3.2. If S is regular and intra-regular Γ-Semihyperring, then $Q \cap R \cap L \subseteq L\Gamma Q\Gamma R$, for any quasi-ideal Q, a right ideal R and a left ideal L of S.

Proof. Let S be regular and an intra-regular Γ-Semihyperring and $x \in Q \cap R \cap L$, where Q be a quasi-ideal, R be a right ideal and L be a left ideal of S. Then we have, $x \in x\Gamma S\Gamma x$ and $x \in S\Gamma x\Gamma x S$ respectively. Therefore

$x\Gamma S\Gamma x \subseteq (S\Gamma x\Gamma xS)\Gamma S\Gamma(S\Gamma x\Gamma xS)$ (Since S an intra-regular)

$= (S\Gamma x)\Gamma(x\Gamma S\Gamma S\Gamma S\Gamma x)\Gamma(x\Gamma S)$

$\subseteq (S\Gamma x)\Gamma(x\Gamma S\Gamma x)\Gamma(x\Gamma S)$ (Since $S\Gamma S\Gamma S \subseteq S$)

$\subseteq (S\Gamma L)\Gamma(Q\Gamma S\Gamma Q)\Gamma(R\Gamma S)$ (Since $x \in Q \cap R \cap L$)

As S is a regular by Theorem 3.1, we have $Q\Gamma S\Gamma Q = Q$. Also $S\Gamma L \subseteq L$, sincs L is a left ideal and $R\Gamma S \subseteq R$ as R is a right ideal. Hence $(S\Gamma L)\Gamma(Q\Gamma S\Gamma Q)\Gamma(R\Gamma S) \subseteq L\Gamma Q\Gamma R$. Therefore $x\Gamma S\Gamma x \subseteq L\Gamma Q\Gamma R$. Hence $x \in x\Gamma S\Gamma x$ and $x\Gamma S\Gamma x \subseteq L\Gamma Q\Gamma R$. Thus $x \in Q \cap R \cap L$ implies $x \in L\Gamma Q\Gamma R$. Thus we get,$Q \cap R \cap L \subseteq L\Gamma Q\Gamma R$.

Theorem 3.3. If S is regular and intra-regular, then $B \cap R \cap L \subseteq L\Gamma B\Gamma R$, for any quasi-ideal B, a right ideal R and a left ideal L of S.

Proof. By Theorems 2.9. and 3.2. Proof follows easily.

Theorem 3.4. Following statements are equivalent in Γ-Semihyperring S.

1. S is regular and an intra-regular
2. For each bi-ideal B of S, $B = B^2 = B\Gamma B$.
3. For each quasi-ideal Q of S, $Q = Q^2 = Q\Gamma Q$.

Proof. (1) ⇒ (2)

Suppose S is regular and intra-regular Γ-Semihyperring. For any bi-ideal B of S, let $x \in B$. As S is regular and intra-regular, we have $x \in x\Gamma S\Gamma x$ and $x \in S\Gamma x\Gamma x\Gamma S$ respectively. Then

$x\Gamma S\Gamma x \subseteq x\Gamma S\Gamma(x\Gamma S\Gamma x)$ (Since S is regular)

$\subseteq x\Gamma S\Gamma(S\Gamma x\Gamma x\Gamma S)\Gamma S\Gamma x$ (Since S is intra-regular)

$= (x\Gamma S\Gamma S\Gamma x)\Gamma(x\Gamma S\Gamma S\Gamma x)$

$\subseteq (x\Gamma S\Gamma x)\Gamma(x\Gamma S\Gamma x)$ (Since $S\Gamma S \subseteq S$)

$\subseteq (B\Gamma S\Gamma B)\Gamma(B\Gamma S\Gamma B)$ (Since $x \in B$)

$\subseteq B\Gamma B$ (Since B is a bi-ideal)

Hence we get, $x \in x\Gamma S\Gamma x \subseteq B\Gamma B$. Thus we get, $B \subseteq B\Gamma B$. As $B\Gamma B \subseteq B$ always holds, we get $B = B\Gamma B$.

(3) ⇒ (4) Every quasi-ideal is a bi-ideal by Theorem 2.9. So implication easily follws.

(3) ⇒ (1)

Let L be a left ideal and R be a right ideal of Γ-Semihyperring S. Then $R \cap L$ is a quasi-ideal of S by Theorem 2.16. Hence by (4), we have $R \cap L = (R \cap L)^2 = (R \cap L)\Gamma(R \cap L) \subseteq L\Gamma\mathrm{R}$. Thus we get, $R \cap L \subseteq L\Gamma\mathrm{R}$. Thus by Theorem 2.13. S is intra-regular Γ-Semihyperring. Also we have $R \cap L = (R \cap L)^2 = (R \cap L)\Gamma(R \cap L) \subseteq R\Gamma\mathrm{L} \subseteq R \cap L$. Therefore $R \cap L = R\Gamma\mathrm{L}$. Hence by Theorem 2.15. S is a regular Γ-Semihyperring.

Theorem 2.2. Following statements are equivalent in S.

1. S is regular and an intra-regular
2. For every bi-ideals B_1 and B_2 of S $B_1 \cap B_2 \subseteq (B_1\Gamma B_2) \cap (B_2\Gamma B_1)$.
3. For every bi-ideal B and quasi-ideal Q of S, $B \cap Q \subseteq (Q\Gamma B)\bigcap(B\Gamma Q)$.
4. For every bi-ideals Q_1 and Q_2 of S $Q_1 \cap Q_2 \subseteq (Q_1\Gamma Q_2) \cap (Q_2\Gamma Q_1)$.

Proof. (1) ⇒ (2)

Suppose S is regular and intra-regular. Let $x \in B_1 \cap B_2$, for any two bi-ideals B_1 and B_2 of S. As S is regular and intra-regular, we have $x \in x\Gamma S\Gamma x$ and $x \in S\Gamma x\Gamma x\Gamma S$ respectively. Then

$x\Gamma S\Gamma x \subseteq x\Gamma S\Gamma(x\Gamma S\Gamma x)$ (Since S is regular)

$\subseteq x\Gamma S\Gamma(S\Gamma x\Gamma x\Gamma S)\Gamma S\Gamma x$ (Since S is intra-regular)

$= (x\Gamma S\Gamma S\Gamma x)\Gamma(x\Gamma S\Gamma S\Gamma x)$

$\subseteq (x\Gamma S\Gamma x)\Gamma(x\Gamma S\Gamma x)$ (Since $S\Gamma S \subseteq S$)

$\subseteq (B_1\Gamma S\Gamma B_2)\Gamma(B_1\Gamma S\Gamma B_2)$ (Since $x \in B$)

$\subseteq B_1\Gamma B_2$ (Since B is a bi-ideal)

Hence we get, $x \in x\Gamma S\Gamma x \subseteq B_1\Gamma B_2$. Similarly $x \in B_2\Gamma B_1$Thus we get, $B_1 \cap B_2 \subseteq B_1\Gamma B_2$ and $B_1 \cap B_2 \subseteq B_2\Gamma B_1$. Therefore we get, $B_1 \cap B_2 \subseteq (B_1\Gamma B_2) \cap (B_2\Gamma B_1)$.

(2) ⇒ (3), (3) ⇒ (4)

As every quasi-ideal is a bi-ideal, implications hold.

(4) ⇒ (1)

Let L be a left ideal and R be a right ideal of a Γ-Semihyperring S. By Theorem 2.11. R and L are quasi-ideals of S. Therefore by (4), we have $R \cap L \subseteq (R\Gamma L) \cap (L\Gamma\mathrm{R}) \subseteq L\Gamma\mathrm{R}$. But Thus we get, $R \cap L \subseteq L\Gamma\mathrm{R}$. Thus by Theorem 2.13. S is intra-regular Γ-Semihyperring. Also, $R\Gamma\mathrm{L} \subseteq R \cap L$ holds always. Therefore we get, $R \cap L = R\Gamma\mathrm{L}$. Hence by Theorem 2.15. S is a regular Γ-Semihyperring.

4. Conclusion

While studying different concepts of classical algebraic structure in hyperstructure theory we found that there is lot of scope for study in hyperstructure theory. In present paper we found that the result studied for Γ- semirings in [2] are holds in Γ-Semihyperring. We make conclusion that there is scope for more results in intra-regular and regular Γ-Semihyperring.

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